# Measuring Luminance with a Digital Camera

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## Contents

# 1 Introduction

There is growing awareness of the problem of light pollution, and with that an increasing need to be able to measure the levels and distribution of light. This paper shows how such measurements may be made with a digital camera.

Light measurements are generally of two types: *illuminance* and *luminance*.

Illuminance is a measure of the light falling on a surface, measured in lux. Illuminance is widely used by lighting designers to specify light levels. In the assessment of light pollution, horizontal and vertical measurements of illuminance are used to assess light trespass and over lighting.

Luminance is the measure of light radiating from a source, measured in candela per square meter. Luminance is perceived by the human viewer as the *brightness* of a light source. In the assessment of light pollution, luminance can be used to assess glare, up-light and spill-light[1].

A detailed explanation of of illuminance and luminance is in [1]. The units used in measurement of light are also summarized in section 3.1 on page 11.

An illuminance meter is an inexpensive instrument, costing about $60. See for example the Mastech LX1330B [2], figure 1(a). A luminance meter is a much more expensive device. For example the Minolta LS-100 Luminance Meter shown in figure 1(b) costs about $3500 [3]. Both measurements are useful in documenting incidents of light pollution but luminance measurements are less common in practice – understandably, give the cost of the instrument.

The pixel values in an image from a digital camera are proportional to the luminance in the original scene. so a digital camera can act as a luminance meter. In effect – providing they can be calibrated – each of the millions of pixels in the light sensor becomes a luminance sensor.

There are significant advantages using a digital camera for measurement of luminance [5]:

- A digital camera captures the luminance of an entire scene. This speeds up the measuring process and allows multiple measurements at the same instant.
- The surroundings of the luminance measurement are recorded, which puts the measurement in context.
- For luminance measurement, the field of view (FOV) of the sensor must be smaller than the source. The FOV of a luminance meter is about 1°. The FOV of a digital camera pixel is on the order of 150 times smaller, so it can measure small area light sources such as individual light emitting diodes. These sources are difficult or impossible to measure with a luminance meter.


(a) Lux meter

(b) Luminance Meter


(c) Digital Camera: Canon SX120IS

Figure 1: Light Measuring Devices

By photographing a source of known luminance, one obtains the conversion factor that links luminance (in candela per square meter) to the value of a pixel in an image. Consequently, the key to calibration is a light source of known luminance. In the following section, we describe the selection of a luminance standard for camera calibration. Then we describe the calibration of the camera, the interpretation of the image data and an example. The appendices elaborate on some of the topics.

[1] Up-light is wasted light that contributes to sky glow. *Spill light* refers to unnecessary light that is produced from a building or other structure, that contributes to over-lighting and bird strikes.

## 2 Luminance Standard

Over a period of some 18 months, we investigated a variety of techniques for creating a luminance standard, as summarized in figure 2.

| **Technique** | **Comments** |
|---|---|
| Comparison with Standard | Luminance standards are very expensive.<br>Laboratory calibration is expensive ($1800).<br>Luminance meter is very expensive ($3500). |
| Natural sources: Sun, Moon | Sun luminance is predictable.<br>Moon luminance is somewhat predictable but surface brightness is uneven.<br>Atmospheric extinction is variable and difficult to predict. |
| Surface Brightness, Frosted Lamp | Relies on known polar distribution of illumination.<br>Distribution of luminance is difficult to predict.<br>Nameplate output in lumens is often inaccurate. |
| Tungsten Filament | Luminance is predictable, but a very sensitive function of temperature.<br>Difficult to make accurate.<br>Effective area of *coiled-coiled* filament is difficult to predict. |
| Illuminance-Luminance | Illuminance measurement can be accurate.<br>Reflectance of surface must be known.<br>Requires diffuse illumination of surface, which is difficult to achieve. |
| Integrating Sphere Standard | Reflectance of sphere interior must be known.<br>Port luminance is proportional to port illuminance.<br>Luminance of port is predictable and consistent.<br>Port image can be used as *flat field*. |
| Photographic Spotmeter | Same field of view as luminance meter ($1^\circ$).<br>Order of magnitude lower cost than luminance meter.<br>Lower resolution than luminance meter (6% vs 1%). |

Figure 2: Summary of Calibration Techniques

Now we discuss each of these in some detail.

**Comparison with Standard**

Professional caliber luminance meters and laboratory-calibrated luminance sources are far too expensive for general purpose use by those working in light pollution abatement. For example, the new cost of a Minolta LS-100 is $3500. The National Research Council of Canada quoted $1800 to calibrate a luminance meter[2].

**Natural Sources**

Some natural sources (the sun, moon and stars) have a predictable luminance. The problem, as detailed in section 3.7, is *atmospheric extinction*, the attenuation of light in the atmosphere, which is not easily determined with precision. The luminance of clear or overcast sky is variable over time and viewing angle [6].

[2] There are some bargains to be had. The author obtained a Tek J-16 Photometer with various measuring heads, including the J6503 luminance probe, from eBay [8]. However, the unit required some repair work and the age of the unit (1985) suggested that the calibration was not reliable.

A *standard candle* illuminates white paper at 1 ft-lambert (3.44 cd/m$^2$) [7]. Unfortunately, readily available candles have significant differences in light output and vary substantially over time [9].

### Surface Brightness of Frosted Lamp

It is possible to relate the total output flux of a lamp (in lumens, specified on the package) to the brightness of the surface of the lamp. For this to work, one must have an accurate map of the radiation pattern. This would be straightforward if the lamp had a spherical radiation pattern, but that's usually not the case. The luminance at the top of the bulb, or at the side in line with the filament, is quite noticeably less than other areas of the bulb. Furthermore, the nameplate flux may be significantly different than the actual flux.

Another approach is to measure the illuminance at some distance from the lamp and then calculate the luminance of the source. Again, that approach requires accurate knowledge of the radiation pattern.

The details and an example are in section 3.6.

### Tungsten Filament

The luminance of the tungsten filament in an incandescent lamp is a predictable function of its temperature. The filament temperature can be determined from its resistance, which in turn depend on the operating current and voltage. In theory, this makes the basis a luminance standard [10].

This approach is attractive because the filament temperature can be related to the luminous output of the lamp, which is a useful check on the calculations.

There are two complications. First, the luminance is a 9th power function of filament temperature. A 1% error in temperature results in a 9% error in luminance. Consequently, the accuracy requirement for temperature is very stringent. Second, the luminance may be affected by a *light recycling* effect resulting from the complex *coiled-coiled* shape of the filament.

The tungsten filament is a small target. It does not fill the entire aperture of a luminance meter such as the Tektronix J-16 with 6503 probe. This would require a correction factor, another possible source of error. On the other hand, a digital camera could be used to record the luminance of a filament, assuming that the area of the filament completely covers several pixels.

A tungsten filament could be useful as a luminance standard, but it needs to be verified by other methods.

### Illuminance to Luminance

Under the right circumstances, the luminance $L$ of a surface is related to the illuminance $E$ and reflectance $\rho$ by equation 1.

$$L = \frac{E\rho}{\pi} \text{ candela/meter}^2 \tag{1}$$

where the quantities are

| | | |
|---|---|---|
| L | Luminance emitted from the surface | candela/meter$^2$ |
| E | Illuminance of light falling on the surface | lux |
| $\rho$ | Reflectance | (dimensionless) |

The device used to measure $E$ – an illuminance meter or luxmeter – is readily available and inexpensive. Determining the reflectance of a surface is a bit more complicated [11], [12], but it can be done. The grey card popular in photography – readily available at low cost – has a known reflectance of 18% and can be used as a standard for comparison.

The challenge in this method is properly illuminating the surface. It must be illuminated uniformly and equally from all directions. In other words, the illuminating field must be *diffuse*. It turns out that this is very difficult to achieve in a laboratory setting with conventional lighting sources. However, a diffuse field *is* achievable inside an integrating sphere, as described below, and under those circumstances this becomes a practical technique.

### Integrating Sphere

An *integrating sphere* can be used as a standard. Figure 3 shows the 14 inch diameter light integrating sphere that was constructed for these measurements. Construction and use of the sphere is described in reference [13].

In operation, one measures the illuminance of the light field exiting the port. (Notice the illuminance meter to the left of the sphere in figure 3.) The reflectance of the interior surface of the sphere is known. Then the luminance of the light field exiting the port is determined by equation 1. In figure 3, the Tektronix J-16 Photometer with 6503 Luminance Probe is located at the sphere port to measure the luminance. One could then calibrate the luminance meter to the calculated value of luminance.

Figure 3: Light Integrating Sphere

The measurement is dependent on the sphere functioning correctly, that is, providing a diffuse field of light. That the field is diffuse can be easily determined from measurements at the port.

- The accuracy of the method depends on the accuracy of the luxmeter and the figure used for reflectance. There are very few error sources compared to other techniques.
- The port provides a large area of uniform illumination. It is a suitable target for luminance meter (because it fills the entire measurement aperture) or as the flat field image for a digital camera.

### Photographic Spotmeter

A *photographic spotmeter* is a narrow field of view exposure meter used in photography. The spotmeter is available from a number of manufacturers. We'll focus on the Minolta Spotmeter F, which is similar in appearance to the Minolta LS-100 luminance meter shown in figure 1(b) on page 2.

A photographic spotmeter displays *exposure value* [14]. The exposure value is the degree of exposure of the camera film or sensor. Various combinations of aperture and shutter speed can be used to obtain the same exposure value, according to equation 2.

$$2^{\mathrm{EV}} = \frac{N^2}{t} \tag{2}$$

where the quantities are:

| | |
|---|---|
| EV | Exposure Value |
| $N$ | Aperture number (F-Stop) |
| $t$ | Exposure time, seconds |

It can also be shown [15] that scene luminance and exposure value are related according to equation 3.

$$2^{\mathrm{EV}} = \frac{L_s\, S}{K_m} \tag{3}$$

where the quantities are:

| | |
|---|---|
| EV | Exposure Value, as before |
| $S$ | ISO setting (see section 3.8, page 17) |
| $L_s$ | Scene Luminance, candela/meter$^2$ |
| $K_m$ | Calibration constant for the meter, equal to 14 for Minolta |

Equation 3 provides us with a route to measuring luminance. Typically, the spotmeter displays EV units at an assumed ISO of 100. A written table in the meter operating manual or software in the unit incorporates the meter constant (14 for Minolta) and uses equation 3 to convert exposure value to luminance.

To get some idea of the advantages and limitations of a photographic spotmeter, it's worthwhile to compare the Minolta LS-100 luminance meter [16] and Minolta F photographic spotmeter [17].

| | **Minolta LS-100 Luminance Meter** | **Minolta F Spotmeter** |
|---|---|---|
| Field of View | 1 degree | 1 degree |
| Range | 0.001 to 300,000 cd/meter$^2$ | 0.29 to 831,900 cd/meter$^2$ |
| Accuracy | 2% | 7% |
| Resolution | 0.1% | 6% |
| Price | $3500.00 (New) $2500 (Used) | $339.00 (Used) |

The resolution and accuracy of the spotmeter are more than satisfactory for photographic use, but rather coarse for for a precise measurement of luminance. That said, it may be useful to rent a photographic spotmeter for a modest fee[3], in order to do a sanity check on a luminance calibration source.

If you have access to a spotmeter, does that eliminate the need for a calibration source such as the integrating sphere? It depends on the required accuracy.

We purchased a used Minolta M spotmeter. When we measured the integrating sphere port luminance, we found that the luminance reading corresponded to an unrealistically value of reflectance in the integrating sphere, 90%. A direct measurement of reflectance – and other methods – indicate a reflectance in the order of 77%. Adjusting the spot meter by 0.3 EV units made the calculated reflectance consistent with other measurements. An adjustment of 0.3 EV units is equivalent to a 23% change in luminance reading. For accuracy better than that, some form of calibration standard such as an integrating sphere is necessary.

# 3 Camera Calibration

## Luminance to Pixel Value

The digital camera turns an image into a two dimensional array of pixels. Ignoring the complication of colour, each pixel has a value that represents the light intensity at that point.

The amount of exposure (the brightness in the final image) is proportional to the number of electrons that are released by the photons of light impinging on the sensor[4]. Consequently, it's proportional to the illuminance (in lux) times the exposure time, so the brightness is in lux-seconds. Invoking the parameters of the camera, we have in formula form [15]:

$$N_d = K_c \left( \frac{t\,S}{{f_s}^2} \right) L_s \tag{4}$$

where the quantities are

| | |
|---|---|
| $N_d$ | Digital number (value) of the pixel in the image |
| $K_c$ | Calibration constant for the camera |
| $t$ | Exposure time, seconds |
| $f_s$ | Aperture number (f-stop) |
| $S$ | ISO Sensitivity of the film (section 3.8, page 17) |
| $L_s$ | Luminance of the scene, candela/meter$^2$ |

The digital number (value) $N_d$ of a pixel is determined from an analysis of the image, using a program like

[3]In Toronto, Vistek will rent a Minolta F spotmeter for $15 per day.

[4]Clark [4] calculates for the Canon D10 DSLR camera at ISO 400, that each digital count in a pixel value is equivalent to 28.3 photons.

*ImageJ* [18]. Pixel value is directly proportional to scene luminance $L_s$. It's also dependent on the camera settings.

For example, if the luminance is constant while the exposure time or film speed are doubled, the pixel value should also double. If the aperture number $f_s$ is increased one stop (a factor of 1.4), the area of the aperture is reduced by half so the pixel value will also drop by half [19].

In theory, to calibrate the camera one photographs a known luminance, plugs values for luminance, exposure time, film speed and aperture setting into equation 4, and calculates the calibration constant $K_c$.

It should then be possible to use the camera at other settings of exposure time, film speed and aperture setting. One determines the pixel value in the image and then runs equation 4 in the other direction to calculate an unknown luminance.

### Maximum Pixel Value

The pixel values are represented inside the camera as binary numbers. The range for the pixel value $N_d$ is from zero to $N_{max}$, where:

$$N_{max} = 2^B - 1 \tag{5}$$

where $B$ is the number of bits in the binary numbers. For example, for a 16 bit raw image, the range of values is from zero to $2^{16} - 1 = 65535$. For an 8 bit JPEG image, the range of values is considerably smaller, from zero to $2^8 - 1 = 255$. In order not to lose information in the image the exposure must be adjusted so that the maximum pixel value is not exceeded.

### Vignetting

The light transmission of the camera lens tends to decrease toward the edges of the lens, an effect known as *vignetting*. The effect can be quantified in equation form. However it is more practical to photograph an image with uniform brightness (a so-called *flat field*). Then use an image analysis program to check pixel value near the centre and near the edge[5] The exit port of the integrating sphere used for these measurements [13] is a suitable flat field. It's illumination has been measured with a narrow field luminance meter and determined to be reasonably uniform.

## Image File Format: Raw, DNG, JPEG and TIFF

Image formats fall into two categories: raw (lossless) and compressed (lossy).

A *raw* image file stores the pixel values exactly as they are generated by the image sensor. Each camera manufacturer uses a different raw format and image processing programs in general cannot accept proprietary format files. Consequently, it is usual to convert raw format to some more universal format, such as TIFF (tagged image file format). A TIFF formatted file can contain all the information of the original so it can also be a lossless format[6].

Raw and TIFF have the advantage that no information is lost, but the files are very large. For example, a Canon raw format (DNG) is typically about 15 MBytes per image. A TIFF formatted file is larger, in the order of 60 MBytes per image. These are colour images. If a TIFF file is processed to monochrome (which is the case for luminance measurements), then the file size drops to about 20MBytes per image.

It turns out that raw format images are overkill for many applications. There is redundant information in most images and with careful processing, the numerical representation of each pixel value can be reduced from 16 bits to 8 bits. To a human observer, there is little or no difference between raw and compressed versions of an image. Compressing an image is *lossy* – the process cannot be reversed to reconstitute the original raw image. However, the size is reduced tremendously which saves on storage space and speeds up image transfer. A typical JPEG-compressed image file is about 71 KBytes in size, a factor of 800 smaller than the comparable TIFF image.

[5] In the case of the Canon SX-120IS point-and-shoot camera, it appears that vignetting is absent, possibly by compensation by the camera computer.

[6] The TIFF file format has the capability of using no compression, lossless compression and lossy compression. It's frequently used in lossless mode.

Compressed images permit many more images in a given storage space and transfer more quickly between camera and computer.

High end digital cameras such as digital single-lens reflex (DSLR) cameras can produce images in raw or compressed (JPEG) format. Most point-and-shoot cameras can only produce compressed format[7].

### Image File Format and Luminance Measurement

A JPEG formatted image has a non-linear relationship between exposure value and pixel code. In calibrating a camera for luminance measurement is necessary to determine this relationship and account for its effect under conditions of different aperture, exposure interval, and ISO number. This greatly complicates the analysis.

A raw formatted image, on the other hand, uses equation 4 (page 4) directly. Image pixel value is directly or inversely proportional to the camera parameters and the scene luminance. For example, a plot of pixel value vs exposure interval is a straight line that passes through the origin.

In theory, both raw and compressed format images can be used for luminance measurement. Jacobs [21], Gabele-Wüller [23], [5] used JPEG formatted images. Craine [22] and Flanders [26] used JPEG format images, but restricted the exposure range to minimize the non-linearity of JPEG compression. Meyer [24] and Hollan [25] used raw format.

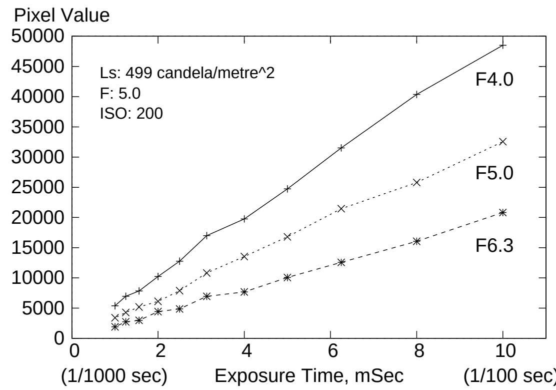
Initially we worked with JPEG formatted images and then subsequently switched to raw format. Raw format images simplified the process and generated results that were more predictable and consistent.

## Settings and Measurements

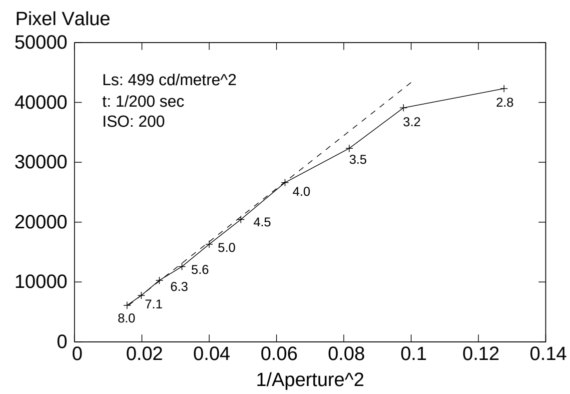
In the ideal case, the camera calibration relationship (equation 4 on page 6) would apply exactly to the camera. In that case, one measurement would be sufficient to determine the value of the calibration constant. One would photograph some source of known luminance $L_s$, determine the equivalent value $N_d$ of the pixels in the image, and note the camera settings for ISO, exposure time and aperture. Plug these values into equation 4 and solve for the calibration constant $K_c$.

For that approach to work, the settings for shutter speed, aperture and ISO must be reflected accurately in the operation of the camera hardware.

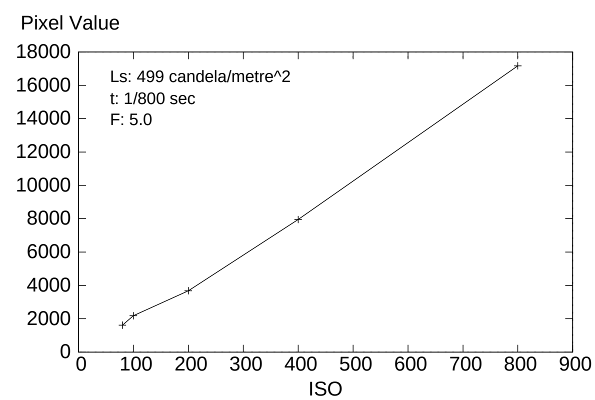
Figure 4(a) shows this is the case for camera shutter speed, where pixel value increases linearly with exposure time (inversely with shutter speed)[8].




(a) Pixel Value vs Exposure Time




(b) Pixel Value vs Aperture




(c) Pixel Value vs ISO

Figure 4: Canon SX120IS Camera Calibration

[7] Canon point-and-shoot cameras can be modified with the so-called CHDK software [20], which enables them to produce raw format images.

[8] Debevec [27] has an interesting comment on shutter speed: *Most modern SLR cameras have electronically controlled shutters which give extremely accurate and reproducible exposure times. We tested our Canon EOS Elan camera by using a Macintosh to make digital audio recordings of the shutter. By analyzing these recordings we were able to verify the accuracy of the exposure times to within a thousandth of a second.*

*Conveniently, we determined that the actual exposure times varied by powers of two between stops ( 1/64, 1/32, 1/16, 1/8, 1/4, 1/2, 1, 2, 4,*

Figure 4(b) shows the relationship between pixel value and aperture. At large apertures, above F 4.0, the relationship becomes non-linear. One would either avoid those aperture values or determine the specific value for the calibration constant at each aperture.

Figure 4(c) shows the relationship between pixel value and ISO[9]. As expected, the digital number is a linear function of the ISO over the range shown on the graph[10].

With a large number of measurements in hand at various values of shutter speed, aperture and ISO, excluding values where the relationship is non-linear, we used a spreadsheet to solve for the corresponding value of the camera constant $K_c$ in equation 4. For the SX120IS camera, using 55 different combinations of settings we measured a camera constant value $K_c$ of 815 with an RMS deviation[11] of 4.7%.

### Magnification

Many sources of light pollution form a small camera image, so it is very useful to be able to magnify the image using the camera lens zoom function. The Canon SX120IS used in this exercise has a zoom range of $\times 1$ to $\times 10$. The CHDK software extends this to $\times 23$. (Also helpful in the case of the Canon SX120IS is the electronic image stabilization feature, which allows one to take hand-held pictures at the maximum zoom setting).

If the camera zoom lens setting is changed, does that have an effect on the luminance measurement? At first thought that would seem to be the case, since luminance is measured in candela per square metre, and the area of observation has changed. However, the luminous power in candela is measured in lumens per steradian (solid angle). A change of magnification alters the area and solid angle such that the two effects cancel. In an ideal system, where there are no losses, luminance is invariant [28], [29] .

We checked this by taking images of the integrating sphere port from a distance of 6.25 meters, at zoom settings of $\times 1$, $\times 10$ and $\times 23$. The average digital number in the image of the illuminated sphere port was constant for the three images, to within 2%.

This is very convenient, since the same camera calibration constant applies regardless of the camera zoom lens setting.

## 4 Example Measurement: LED Array

We use the light emitting diode (LED) array of figure 5(a) to illustrate the process of using a digital photograph to measure the luminance of the individual LEDs. This cannot be done easily using the Tek J-16 luminance meter or Minolta Spotmeter M because their sensor field of view is much larger than the light emitting source, and a measurement will underestimate the luminance.

The array was photographed with the camera in Manual mode to establish the shutter speed, aperture and ISO, keeping an eye on the camera real-time histogram display to ensure that the image was not overexposed. The camera CHDK software generates two images, a Canon RAW format image and a JPEG formatted image. The RAW format image was processed to monochrome TIFF format, as described in *Work flow*, section 3.9, page 18.

An image of the illuminated array is shown in figure 5(b). The TIFF version of the image was analysed in *ImageJ*. A profile line was drawn through three of the LEDs to determine the corresponding pixel values, using the profile analysis tool. The resultant plot is shown in figure 5(c).

The EXIF file from the JPEG image was used to confirm the camera settings. The camera constant was determined earlier, page 9. The image information is summarized as follows:

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*8, 16, 32), rather than the rounded numbers displayed on the camera readout ( 1/60, 1/30, 1/15, 1/8, 1/4, 1/2, 1, 2, 4, 8, 15, 30).*

[9] ISO number is referred to as *film speed* in film cameras. In a digital camera, it is a function of the amplification applied to the pixel value after capture.

[10] This camera provides an additional ISO setting of 1600, but the increase in digital number for that ISO setting is not proportional at all, which makes the setting unsuitable for luminance measurement. In light pollution work, the intensity of sources makes it unlikely that ISO 1600 would be useful. It could however be of interest to someone documenting sky glow, which is relatively faint.

[11] RMS: Root Mean Square. The deviation values are first squared. Then one takes the average of these squares. Finally, one takes the square root of the average. The result is an indication of the typical value of a deviation, while ignoring the effect of the sign of the deviation.

| Quantity | Symbol | Value |
|---|---|---|
| Maximum Pixel Value | $N_d$ | 64197 |
| Shutter Speed | $t$ | 1/2500 sec |
| Aperture (F-Stop) | $f_s$ | 8.0 |
| ISO Setting | $S$ | 80 |
| Camera Constant | $K_c$ | 815 |

Rearranging equation 4 (page 6) to solve for luminance, we have:

$$\begin{aligned} L_s &= \frac{N_d\, f_s^2}{K_c\, t\, S} \\ &= \frac{64197 \times 8.0^2}{815 \times \dfrac{1}{2500} \times 80} \\ &= 157,538 \text{ candela/metre}^2 \end{aligned}$$

According to the table of luminances in section 3.2 (page 11), 50,000 candela/metre$^2$ is *Maximum Visual Tolerance*. This makes the unshielded LED array a potential source of glare.

### Extending Luminance Range

The pixel value in the previous example is very close to the allowable maximum, 65536 ($2^{16}$). The camera settings are at their limit (fastest shutter speed, minimum aperture, minimum ISO) for the camera to minimize exposure. Is it possible to photograph images of greater luminance?

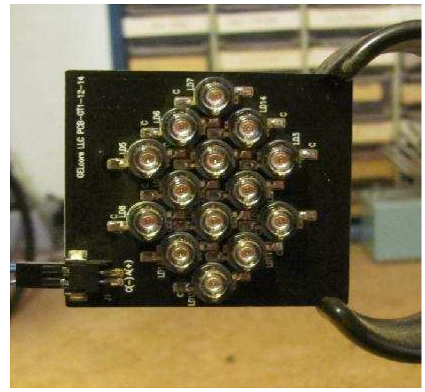
There are two possible solutions.

- The CHDK software on a Canon point-and-shoot camera supports custom exposure settings up to 1/100k seconds [30], [31], so you could increase the shutter speed (ie, decrease the shutter interval). CHDK also supports custom ISO values, starting at 1, so you could reduce the ISO setting.

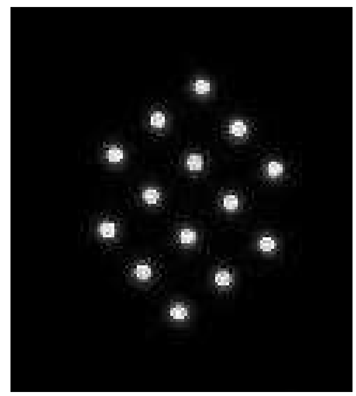
  In general, these options are not available to unmodified DSLR (digital single-lens reflex) cameras.

- A neutral density filter can reduce the luminance of the image. For example, an ND4 filter reduces the luminance by a factor of 4. Neutral density filters are readily available for DSLR cameras.

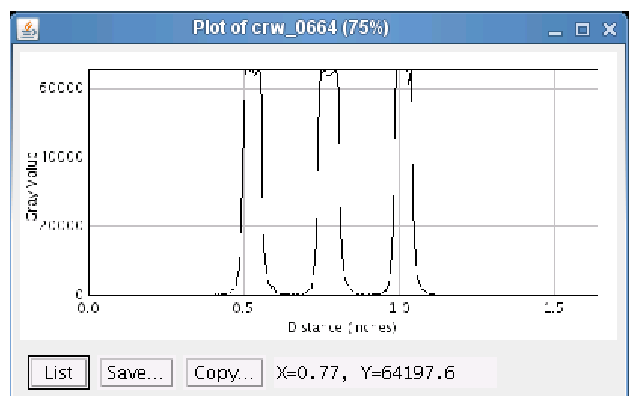
Both methods should be calibrated. With an inexpensive set of neutral density filters we found that the actual attenuation was as much as 30% different from the labeled value.



(a) LED Array



(b) Image




(c) Profile Plot

Figure 5: Light Emitting Diode Array

# 5 Appendices

## 3.1 Light Measurement Symbols and Units

| | | | |
|---|---|---|---|
| Luminous Flux | $\Phi_v$ | Lumen | Luminous power, weighted by the visual response function. |
| Luminous Intensity | $I_v$ | Candela, Lumen/steradian | Luminous power in a particular direction. |
| Illuminance | $E_v$ | Lux, Lumen/metre$^2$ | Density of the luminous flux incident on a plane surface |
| Luminance | $L_v$ | Candela/metre$^2$ | Luminous intensity per unit area, ie, perceived brightness of a source. |
| Luminous Exposure | $H_v$ | Lux-seconds | Time integral of illuminance |
| Luminous Energy | $Q_v$ | Lumen-seconds | Luminous energy, time integral of luminous power |
| Steradian | $\Omega$ | Radians | Solid angle subtending an area $r^2$ at radius $r$. |

## 3.2 Typical Values of Luminance

| **Light Source** | **Luminance, candela per square metre** |
|---|---|
| Sun | $1.6 \times 10^9$ |
| Arc lamp | $1.5 \times 10^8$ |
| Metal halide lamp | $5.3 \times 10^6$ |
| Clear incandescent lamp, filament | $2 \times 10^6$ to $2 \times 10^7$ |
| Frosted incandescent lamp | 50000 to 400000 |
| Low pressure sodium lamp | 75000 |
| Maximum visual tolerance | 50000 |
| Cloud (sunny day) | 35000 |
| Fluorescent lamp | 12000 to 14000 |
| White illuminated cloud | 10000 |
| 60 watt soft-white bulb | 10000 |
| Surface of moon | 1000 |
| Metal-halide flood lamp | 500 |
| Convenience store sign | 150 |
| White paper under lamp | 30 to 50 |
| Television screen (CRT) | 9 |
| Neon lamp | 8 |
| Candle | 7.5 |
| Clear sky | 3 to 5 |
| Moon | 2.5 |
| White paper lit by candle at one foot | 0.29 |
| Dark sky reserve (proposed) | 0.1 |
| Night sky | 0.001 |
| Threshold of vision | 0.000003 |

Sources: [32], [7]

## 3.3 Accuracy of Photometric Measurements

It's common in measurements of electrical quantities (voltage, resistance) to obtain an accuracy in excess of 1%. This level of accuracy is *much* more difficult to achieve in photometric measurements.

- Where there are horizontal and vertical components to the light source, the angle of measurement of the light meter is critical.
- Certain measurements (card reflectance) depend on a large, uniform, diffuse light source [11]. In practice, this can be difficult to create.

- Precision light measurements require a controlled environment. This is not readily available to the non-specialist.
- A photometric light meter (one that responds in a similar fashion to the human eye) contains a spectrum response filter. The spectrum of the source interacts with this curve in such a way that a small error in the filter response may lead to large errors in light level measurement. This is particularly true for sources where the light energy is concentrated at a few discrete wavelengths [33].

The specified and measured accuracies of four illuminance meters is shown in section 3.5 on page 13. With specified accuracies in the order of 5% and measured deviations from the average value in the order of 7%, an overall measurement accuracy of 10% is reasonably achievable[12].

Fortunately, the variability of light level measurements is mitigated to some extent by the non-linear response of the human eye to different light levels, as we document in section 3.4. For example, a 25% change in brightness is *just detectable* by the human vision system.

## 3.4 Perception of Brightness by the Human Vision System

According to Steven's Law [35], brightness increases as a 0.33 power of the luminance[13]. In formula form:

$$S = K\ L^{0.33} \tag{6}$$

where $S$ is the perceived brightness (the *sensation*), $K$ is a constant and $L$ is the luminance.

Since the exponent is less than unity, the equation has the effect of reducing errors in luminance measurement. We illustrate this with an example.

**Example**

Suppose that a luminance measurement is in error by $+10$%. What is the brightness perception of that error?

**Solution**

Call the correct sensation and luminance $S$ and $L$. Call the measured sensation and luminance $S_m$ and $L_m$. Define the ratio $R$: $L_m = R\ L$.

Then from equation 6 we have:

$$\begin{aligned} S &= K\ L^{\ 0.33} \\ S_m &= K\ L_m^{\ 0.33} \\ &= K\ (RL)^{\ 0.33} \\ &= K\ R^{\ 0.33} L^{\ 0.33} \end{aligned} \tag{7}$$

Now find the ratio of the measured and true sensation:

$$\begin{aligned} \frac{S_m}{S} &= \frac{K\ R^{\ 0.33} L^{\ 0.33}}{K L^{\ 0.33}} \\ &= R^{\ 0.33} \end{aligned} \tag{8}$$

The measured luminance is 10% larger than the actual luminance, so $R = 1.10$.

[12] It may be possible to check the calibration of an illuminance meter by measuring the horizontal illuminance from the sun at noon. A table of solar illuminance for various elevations of the sun above the horizon is in reference [34].

[13] This greatly simplifies a complex situation. The perception of brightness is strongly determined by the size of the source and its surroundings. However, it illustrates the concept and is roughly true for a small source against a dark background.

Then:

$$\begin{aligned}\frac{S_m}{S} &= R^{\,0.33} \\ &= 1.10^{\,0.33} \\ &= 1.03\end{aligned}$$

That is, the perception of the brightness is only 3% high when the actual luminance is 10% high.

The *just noticeable difference* for brightness is 7.9%, [36] so the 3% difference would be undetectable.

Using equation 6, one can show that a *just noticeable* change in brightness requires a luminance increase of 25%. A perceived doubling of brightness requires a luminance increase of approximately 8 times (800%).

### 3.5 Comparing Illuminance Meters

To determine the relative consistency of illuminance readings, the readings of four illuminance meters. The meter sensor was placed in exactly the same position each time, and care taken not to shadow the sensor.

| **Meter** | **Range** | **Accuracy** | **Reference** |
|---|---|---|---|
| Tek J10 with J6511 Probe | 0.01 to 19,990 lux | $\pm 5\%$ | [37] |
| Amprobe LM-80 | 0.01 to 20,000 lux | $\pm 3\%$ | [38] |
| Mastech LX1330B | 1 to 200,000 lux | $\pm 3\%$ $\pm 10$ digits | [39] |
| Extech 401025 | 1 to 50,000 lux | $\pm 5\%$ | [40] |

The measurement results are shown in figure 6.

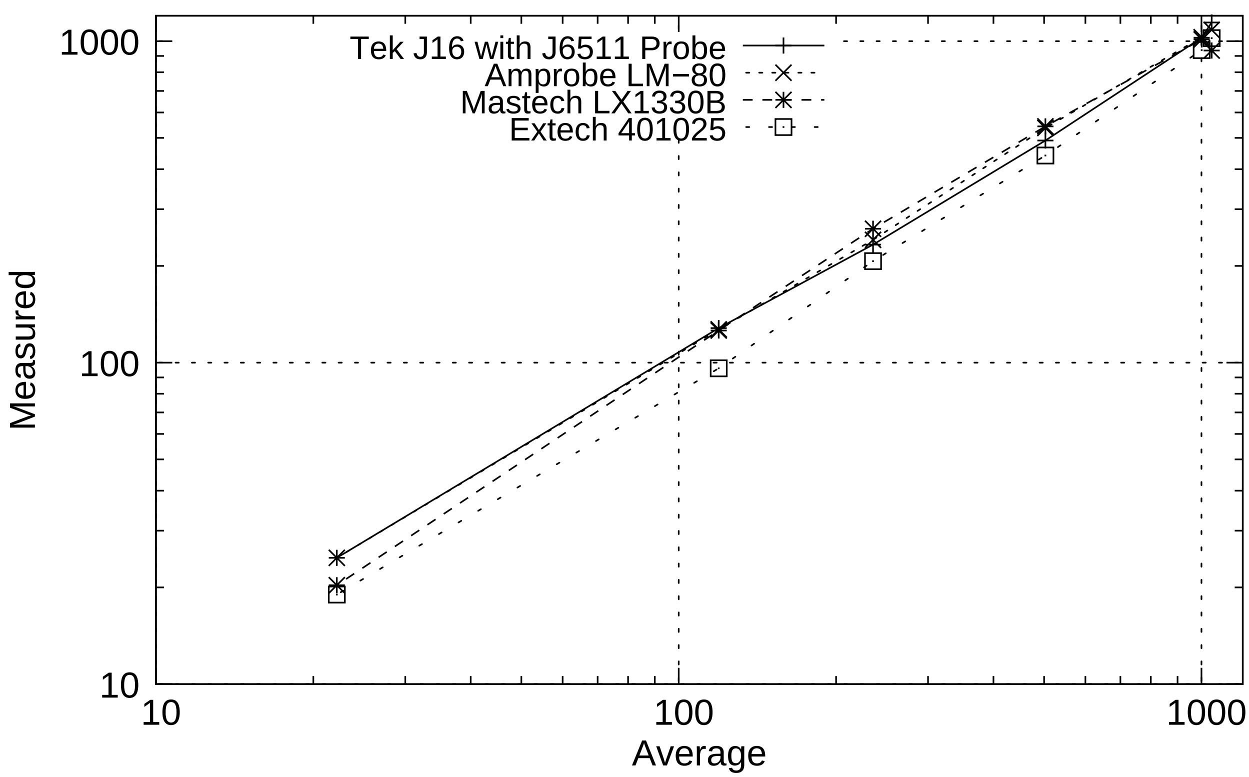



Figure 6: Four Illuminance Meters: Tek J10 with J6511 Probe, Amprobe LM-80, Mastech LX1330B, Extech 401025

The measured RMS deviations from the average were:

| Meter | RMS Deviation from Average |
|---|---|
| Tek J10 with J6511 Probe | 6.9% |
| Amprobe LM-80 | 6.4% |
| Mastech LX1330B | 8.1% |
| Extech 401025 | 12.4% |

On the basis of the specified accuracy and measured deviation from average, an overall measurement accuracy of $\pm 10\%$ is a reasonable goal.

## 3.6 Frosted Incandescent Lamp Calibration

**Luminance from Total Output Flux**

Knowing the total light output in lumens of a frosted incandescent lamp on can in theory determine the surface brightness of the lamp. Referring to the definition of light units (section 3.1 on page 11) we can predict surface luminance as follows:

1. Total luminous flux $\Phi$ in lumens is known from the label on the box.

2. The bulb radiates through some solid angle $\Omega$ steradians.

3. Then the luminous intensity $I$ is given by

$$I = \frac{\Phi}{\Omega} \text{ Candela} \tag{9}$$

4. The luminance of the bulb is equal to the luminous intensity $I$ Candela divided by the surface area of the bulb $A_b$ meters$^2$.

$$L = \frac{I}{A_b} \text{ Candela/meters}^2 \tag{10}$$

**Example: Surface Luminance from Nameplate Output**

The Sylvania DoubleLife 60 watt lamp has a total output of 770 lumens. The bulb diameter is 6cm. What is the luminance of the bulb surface?

**Solution**

1. $\Phi = 770$ lumens.

2. Assume that the radiating pattern is a sphere, with 13% removed to account for the base. Then the radiating angle $\Omega$ is:

$$\begin{aligned} \Omega &= 4 \times \pi \times (1 - 0.13) \\ &= 10.92 \text{ steradians} \end{aligned}$$

3. The luminous intensity $I$ is given by equation 9 above:

$$\begin{aligned} I &= \frac{\Phi}{\Omega} \\ &= \frac{770}{10.92} \\ &= 70.51 \text{ candela} \end{aligned}$$

4. The surface area of the bulb $A_b$ is approximately a sphere of radius $r = 3$ centimeters:

$$\begin{aligned} A_b &= 4\pi r^2 \\ &= 4 \times \pi \times \left(\frac{3}{100}\right)^2 \\ &= 0.0113 \text{ meters}^2 \end{aligned}$$

The luminance is the luminous intensity $I$ divided by the bulb surface area $A_b$ (equation 10 above).

$$\begin{aligned} L &= \frac{I}{A_b} \\ &= \frac{70.51}{0.0113} \\ &= 6240 \text{ candela/meters}^2 \end{aligned}$$

**Measurement Results**

Using the Tektronix J16 photometer with J6503 luminance probe, the measured luminance for this bulb varies from 6100 candela/meters$^2$ at the top of the bulb to 16100 candela/meters$^2$ on the side.

This variation in luminance means that our assumption of a spherical radiation pattern (equal radiation in all directions) is incorrect. An example of a radiation curve for an incandescent lamp (figure 7) confirms that the radiation pattern is not spherical.

Consequently, this technique does not give reliable enough results to be used as a predictable luminance calibration standard.

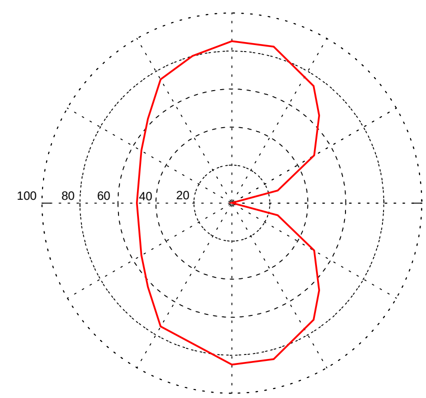


Figure 7: Incandescent Lamp Radiation Pattern. Magnitude is in Candela. Adapted from [41], magnitude scaled to approximate measured data.

**Luminance from Illuminance**

It is also possible to measure the illuminance at some distance from the bulb and extrapolate that to determine the luminous intensity. For completeness we provide the method here. However, it suffers from the same problem as using the nameplate output: the radiation pattern is not known and the assumption of a spherical radiation pattern is incorrect. Consequently, the predicted luminance is not accurate.

The light meter reads light intensity in *lux*, where

$$1 \text{ lux} = 1 \text{ lumen/metre}^2$$

The light energy emitted by the source, in *candela*, is given by

$$1 \text{ candela} = 1 \text{ lumen/steradian}$$

Solid angle in *steradian* is given by

$$\Omega = \frac{A}{r^2}$$

where $\Omega$ is the solid angle in steradians, $r$ is the distance from the source to the surface of an imaginary sphere, and $A$ is an area on the surface of that imaginary sphere.

If we choose the area to be 1 metre$^2$ and the distance $d$ from the source as 1 metre, then the solid angle is 1 steradian. Consequently, the numeric value of the light intensity in lux is equal to the light energy emitted by the source in candela.

If we take the light intensity measurement at some other distance, we can adjust it to 1 metre spacing using the inverse square law.

Once the total light energy output in candela is known, the surface luminance in *candela/m*$^2$ can be calculated, based on the surface area of the lamp.

**Example: Luminance from Illuminance Measurement**

A Sylvania *DoubleLife* 60 watt soft white incandescent lamp produces is measured to produce 273 lux at a distance of 50cm. The bulb can be characterised as a sphere, 6cm in diameter. Determine the luminance of the bulb surface.

**Solution**

1. Adjust the reading in lux to a distance of 1 metre:

$$\begin{aligned} \frac{E_1}{E_2} &= \left(\frac{l_2}{l_1}\right)^2 \\ E_2 &= E_1\left(\frac{l_1}{l_2}\right)^2 \\ &= 273 \times \left(\frac{0.5}{1.0}\right)^2 \\ &= 68.25 \text{ lux, or lumens/m}^2 \end{aligned}$$

2. This is at a distance of 1 metre, so the source is emitting total energy of the same amount, $68.25$ candela (lumen/steradian).

3. The surface area of a sphere is given by $S = 4\pi r^2$, where $r$ is the physical radius of the lamp, 3 centimeters.

$$\begin{aligned} S &= 4\pi r^2 \\ &= 4 \times 3.14 \times \left(\frac{3}{100}\right)^2 \\ &= 0.0113 \text{ meters}^2 \end{aligned}$$

4. Now we can calculate the luminance in candela/metre$^2$.

$$\begin{aligned} L &= \frac{C}{S} \\ &= \frac{68.25}{0.0113} \\ &= 6039 \text{ candela/m}^2 \end{aligned}$$

Light level measurements of the incandescent lamp should be taken under the following conditions:

- Other light sources should be turned off or it should be established that they are dim enough that they do not materially affect the measurement.
- The light source and meter should be arranged so that the direct light from the lamp predominates. That requires reflecting surfaces to be as far away as possible and if necessary cloaked with light absorbing material.
- Ryer [42] suggests setting up an optical bench with a series of baffles. Each baffle is a black opaque sheet mounted at right angles to the optical path. A hole in each baffle is centred on the optical path. This prevents stray light, from a similar direction as the source, from reaching the detector.
- There is a 20% decrease in light output from the side of an incandescent lamp to the top. Consequently, whatever orientation is used for the measurement of light level should also be used for luminance.

### 3.7 Luminance Calibration using Moon, Sun or Daylight

The brightness (luminance) of the sun is a known quantity. Karandikar gives the value[14] as $193,000 \pm 4000$ candles/cm$^2$ [43]. The brightness of the moon is also known, albeit complicated by changing phase [44], [45].

These and other astronomical sources are attractive to use because they have relatively constant luminance. The problem is *atmospheric extinction*, the attenuation of light as it passes through the atmosphere.

The attenuation of light from an astronomical source by the earth's atmosphere is a function two effects. First, the angle of the source above the horizon determines the distance the light travels through the atmosphere. This is predictable. Second, the atmosphere contains more or less atmospheric haze, which affects the attenuation regardless of sun angle. This is known only approximately.

The attenuation of a light source between outer space and ground level is given by Courter [46] as

$$\frac{I}{I^*} = 10^{-0.4 k_e X_m} \tag{11}$$

where

| | |
|---|---|
| $I$ | Intensity at the base of the atmosphere |
| $I^*$ | Intensity at the top of the atmosphere |
| $k_e$ | Extinction coefficient, depending on clarity of the atmosphere, typically 0.20 to 0.27, extreme values 0.11 to 0.7 |
| $X_m$ | Air Mass, equal to $1/\cos(\theta)$, where $\theta$ is the angle between the zenith and the sun angle, 67° in this case. |

Unfortunately, there is no straightforward way to determine the extinction coefficient $k_e$. A variation of 0.11 to 0.7 would a variation from 0.88 to 0.46 in the ratio of extraterrestrial to terrestial luminance, which is unacceptably large.

### 3.8 ISO Speed Rating

In a film camera, the sensitivity of the film (*film speed*) is specified by a so-called ISO number. A faster film (one with a higher ISO number) is more sensitive to light [47]. The commonly used ISO scale for film speed increases in a multiplicative fashion, where the factor is approximately 1.2 between steps[15].

In a digital camera, the ISO number refers to the sensitivity of the image sensor, specifically referred to as the ISO Speed Rating. To change the sensitivity of a film camera, one needs to purchase and install a different film. In a digital camera, the operator (or the automatic exposure mechanism) can set the ISO Speed Rating to a different value[16].

[14] This is an older reference, so the luminance units are given as candles (ie, candela) per square centimeter. Today, we would used candela per square metre.

[15] This arrangement is technically known as the ISO-arithmetic scale, which is similar to its predecessor, so-called ASA film speed scale. There is also an ISO-logarithmic scale, which does not seem to be in common use.

[16] Changing the ISO setting does not alter the behavior of the sensor itself. However, the signal from the sensor is amplified, and changing the amplification factor effectively changes the sensitivity of the camera system. There is a limit to this: at very high gain, the sensor noise becomes prominent.

### 3.9 Work Flow Summary

The image work flow for the Canon SX120IS with CHDK software is shown below. This is completely specific to that camera and configuration. However, a similar work flow may apply to other cameras.

```
Configured to generate RAW files in DNG format
             |
             | file.dng, 12 bit colour
             |
  dcraw -w -4 -T crw_0569.dng
             |
             | file.tiff, 16 bit colour, white balanced
             |
  convert file.tiff -colorspace Gray output.tiff
             |
             | output.tiff, 16 bit black-white
             |
           imagej
  Histogram, profile, average or pseudocolour
```

### 3.10 Processing Scripts

When there are a number of images to be processed in the work flow, it becomes tedious to convert them by hand. The following shell scripts (for the Bash shell under Linux) automate processing the raw images from a Canon SX120IS camera, from the raw colour data to the monochrome image.

Converting the original `crw_xxxx.dng` image to colour TIFF format named `crw_xxxx.tiff` uses the `DCRAW` program [52]:

```
for i in *.dng
do
echo $i
dcraw -w -4 -T $i
done
```

Converting the colour TIFF format to black and white as from `crw_xxxx.tiff` to `output_xxxx.tiff`.

```
for i in crw_*.tiff
do
echo $i
convert $i -colorspace Gray output_${i#crw_}
done
```

### 3.11 Using ImageJ To Determine Pixel Value

1. Start ImageJ.
2. Open the raw image file (eg, `output_0569.tiff`).
3. If the image needs to be scaled, place the cursor in the image and hit either the $+$ or $-$ key to scale the image. This has no effect on the maximum, minimum or average value of the pixels, it just spreads out the image, which may make it easier to work with.
4. Select the area of interest for example with the ellipse tool.
5. Under `Analyse -> Set Measurements` select the measurements required (eg, average grey scale).
6. Select `Analyse -> Measure`. A measurement window pops up or, if the measurements window is already open, adds a line with the measurement of the average pixel value.

Alternatively, you can run a profile line through some area of the image to generate a graph of the pixel values along that line.

1. Select the `Line` tool from the toolbar.
2. Draw the line on the image, for example, through the light source.
3. Select `Analyse -> Plot Profile`.
4. A graph of the pixel value profile appears. You can save the values to a text file for further analysis.

An example plot profile (the port of the integrating sphere) is shown in figure 8.

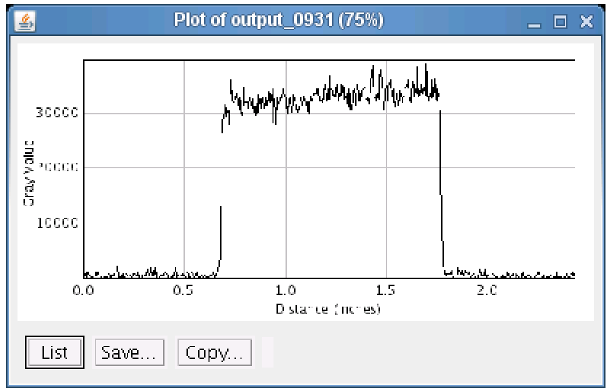



Figure 8: Example Plot Profile

### 3.12 Using ImageJ To Generate a Luminance-Encoded Image

It is possible to use ImageJ to create a pseudo-coloured image, in which colours represent different levels of luminance. This is particularly useful in assessing the illumination for architectural applications. To create a pseudocolour image:

1. Open an image, which can be uncompressed TIFF format.
2. Choose `Image -> Colour -> Show LUT`. The image is then pseudo-coloured using the default lookup table (LUT), which is 0 to 255 grey levels. Notice that even though the image may have $2^1$6 (65536) levels or whatever, the pseudocolour has only $2^8$ (256). But that should be lots for the human observer.
3. Select `Image -> Lookup Tables` to select the 8 bit pseudocolour scheme. For example, the `16 colours` lookup table shows sixteen different colours corresponding to equal parts of the 8 bit pixel range 0 to 255.
4. Select `Image -> Color -> Show LUT` to see a graphical representation of the lookup table, which may then be extrapolated to 16 bit pixel value and then to luminance value.
5. Select `Image -> Colour -> Edit LUT` to see a widget containing 256 colour boxes, each corresponding to a level in the image, that one can edit to modify the LUT.

### 3.13 EXIF Data

EXIF data (also known as *metadata*) is information that is attached to an image. It contains photo parameters such as exposure interval, aperture and ISO speed rating that is required in automating the conversion of image data to luminance. This is enormously convenient in practice. It enables one to collect a series of images without having to manually record the camera settings. One can return to images after the fact and determine settings from the EXIF data.

From [48]:

> Exchangeable image file format (EXIF) is a specification for the image file format used by digital cameras. The specification uses the existing JPEG, TIFF Rev. 6.0, and RIFF WAV file formats, with the addition of specific metadata tags. It is not supported in JPEG 2000, PNG, or GIF.

On a Linux system, the main EXIF data is obtained in the Gnome Nautilus file viewer by clicking on `Properties -> Image`. The information may be copied and pasted into another document using the standard copy and paste function.

On a Windows system, the EXIF data is obtained by right clicking on a photograph and selecting ’properties’. The information cannot be copied and pasted.

A more complete listing of EXIF data is found using the command line program `exiftool` [49]. For example, the command `exiftool myfile.jpg` lists the EXIF data for myfile.jpg.

EXIF data for the Canon SX120IS is shown below. The EXIF data for this camera is *very* extensive. For other point-and-shoot cameras the data file is greatly abbreviated.

For legibility the EXIF data has been formatted slightly into different sections.

`Image Description`

| | |
|---|---|
| Manufacturer | Canon |
| Model | Canon PowerShot SX120 IS |
| Orientation | top - left |
| x-Resolution | 180.00 |
| y-Resolution | 180.00 |
| Resolution Unit | Inch |
| Date and Time | 2009:12:15 20:48:28 |
| YCbCr Positioning | co-sited |

`Thumbnail Directory`

| | |
|---|---|
| Compression | JPEG compression |
| x-Resolution | 180.00 |
| y-Resolution | 180.00 |
| Resolution Unit | Inch |

`Exif Directory`

| | | | |
|---|---|---|---|
| Exposure Time | 1/15 sec. | Maker Note | 2266 bytes unknown data |
| FNumber | f/2.8 | User Comment | |
| ISO Speed Ratings | 640 | FlashPixVersion | FlashPix Version 1.0 |
| Exif Version | Exif Version 2.21 | Color Space | sRGB |
| Date and Time (original) | 2009:12:15 20:48:28 | PixelXDimension | 3648 |
| Date and Time (digitized) | 2009:12:15 20:48:28 | PixelYDimension | 2736 |
| Components Configuration | Y Cb Cr - | Focal Plane x-Resolution | 16141.59 |
| Compressed Bits per Pixel | 3.00 | Focal Plane y-Resolution | 16094.12 |
| Shutter speed | 3.91 EV (APEX: 3, 1/14 sec.) | Focal Plane Resolution Unit | Inch |
| Aperture | 2.97 EV (f/2.8) | Sensing Method | One-chip color area sensor |
| Exposure Bias | 0.00 EV | File Source | DSC |
| MaxApertureValue | 2.97 EV (f/2.8) | Custom Rendered | Normal process |
| Metering Mode | Pattern | Exposure Mode | Auto exposure |
| Flash | Flash fired, auto mode, red-eye reduction mode. | White Balance | Auto white balance |
| | | Digital Zoom Ratio | 1.00 |
| Focal Length | 6.0 mm | Scene Capture Type | Standard |

`InterOperability Directory`

| | |
|---|---|
| InteroperabilityIndex | R98 |
| InteroperabilityVersion | 0100 |
| RelatedImageWidth | 3648 |
| RelatedImageLength | 2736 |

`Extended Metadata Description`

| | |
|---|---|
| Make | Canon |
| Model | Canon PowerShot SX120 IS |
| Orientation | TopLeft |
| Resolution Unit | Inch |
| ModifyDate | 2009-12-15T08:48:28 |
| YCbCrPositioning | 2 |
| ExposureTime | 1/15 |
| FNumber | 28/10 |
| ISOSpeedRatings | 640 |
| ExifVersion | 0221 |
| DateTimeOriginal | 2009-12-15T08:48:28 |
| DateTimeDigitized | 2009-12-15T08:48:28 |

| | |
|---|---|
| Components Configuration | 1 |
| | 2 |
| | 3 |
| | 0 |
| CompressedBitsPerPixel | 3 |
| ShutterSpeedValue | 125/32 |
| ApertureValue | 95/32 |
| ExposureBiasValue | 0/3 |
| MaxApertureValue | 95/32 |
| Metering Mode | Pattern |
| FocalLength | 6 |
| UserComment | |
| FlashPixVersion | 0100 |
| Color Space | StandardRGB |
| PixelXDimension | 3648 |
| PixelYDimension | 2736 |
| FocalPlaneXResolution | 3648000/226 |
| FocalPlaneYResolution | 2736000/170 |
| Focal Plane Resolution Unit | Inch |
| Sensing Method | OneChipColorAreaSensor |
| File Source Type | DCF |
| Custom Rendered | Normal |
| Exposure Mode | Auto |
| White Balance | Auto |
| DigitalZoomRatio | 1 |
| Scene Capture Type | Standard |
| Exported Locations | |

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